NAG Toolbox for MATLAB

s11ac

1 Purpose

s11ac returns the value of the inverse hyperbolic cosine, $\operatorname{arccosh} x$, via the function name. The result is in the principal positive branch.

2 Syntax

```
[result, ifail] = s11ac(x)
```

3 Description

s11ac calculates an approximate value for the inverse hyperbolic cosine, $\operatorname{arccosh} x$. It is based on the relation

$$\operatorname{arccosh} x = \ln\left(x + \sqrt{x^2 - 1}\right).$$

This form is used directly for $1 < x < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $x \geq 10^k$, $\sqrt{x^2 - 1}$ is equal to $\sqrt{x}$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arccosh} x = \ln 2 + \ln x.$$

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $\mathbf{x} \geq 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The function has been called with an argument less than 1.0, for which $\operatorname{arccosh} x$ is not defined. The result returned is zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x} \times \delta \right|.$$

That is the relative error in the argument is amplified by a factor at least $\frac{x}{\sqrt{x^2 - 1} \operatorname{arccosh} x}$ in the result. The equality should apply if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation it is possible that an extra figure may be lost in internal calculation and round-off. The behaviour of the amplification factor is shown in the following graph:

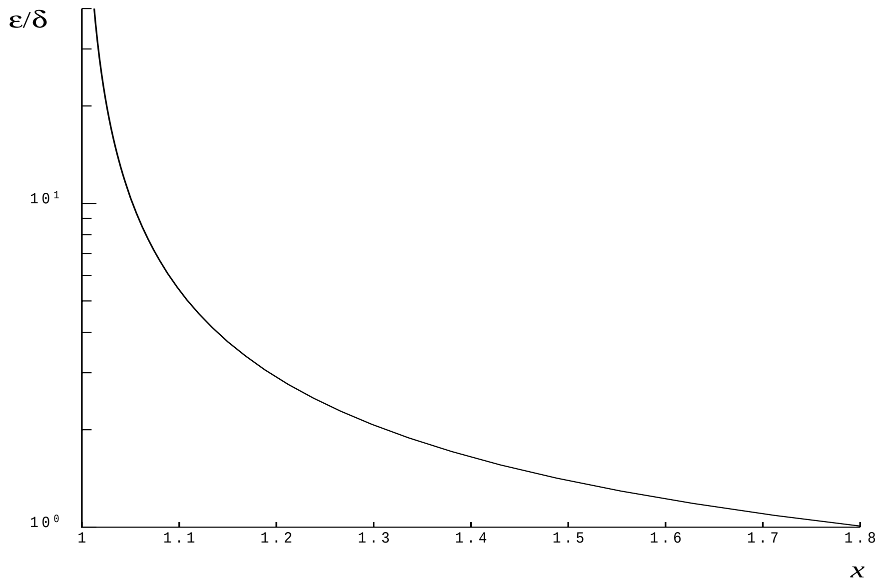

Figure 1

It should be noted that for $x > 2$ the factor is always less than 1.0. For large x we have the absolute error E in the result, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***. More significantly for x close to 1, $x - 1 \sim \delta$, the above analysis becomes inapplicable due to the fact that both function and argument are bounded, $x \geq 1$, $\operatorname{arccosh} x \geq 0$. In this region we have

$$E \sim \sqrt{\delta}.$$

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.

8 Further Comments

None.

9 Example

```
x = 1;
[result, ifail] = s11ac(x)
```

```
result =
     0
ifail =
           0
```